# CIRCLE DEFINITIONS AND THEOREMS

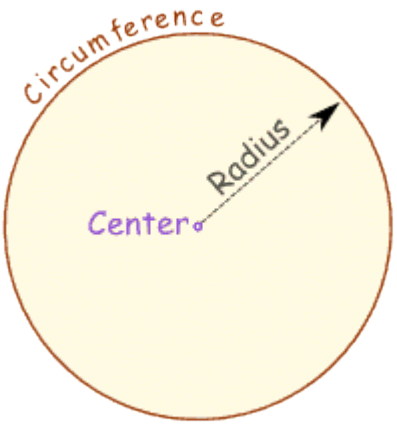
## DEFINITIONS

**Circle-** The set of points in a plane equidistant form a given point(the **center** of the circle).

**Radius-** A segment from the center of the circle to a point on the circle(the distance from the center to a point on the circle.)

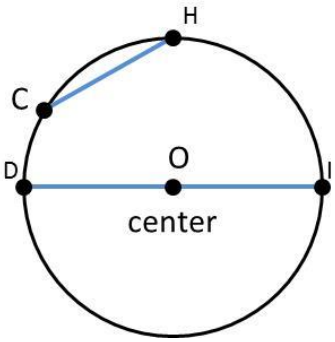
**Circumference –** distance around the edge of the circle

**Congruent Circles-** two circles with the same **radius.**




**Diameter** – A segment that goes through the center of the circle, with both endpoints on the edge of the circle.

**Chord** - A line segment that goes from one point to another on the circle's **circumference.**




$\overline{DI}$ ***is the diameter.***

$\overline{CH}$ ***is a chord.***

$DO$ ***and*** $OI$ ***are both radii.***

$Diameter = 2 * radius.$

$Radius = \frac{1}{2} * diameter$

**Tangent –** a line that intersects a circle at only one point. The radius at the point of tangency is **perpendicular** to the tangent line.

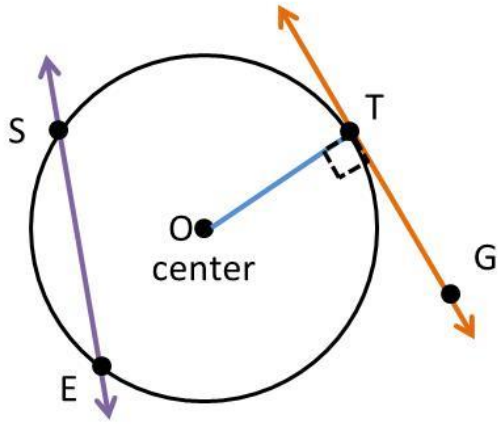
$\overleftrightarrow{TG}$ ***is a tangent line.***

***The point of tangency is the point T.***

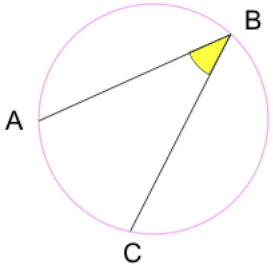
$\overline{OT}$ ***is a radius and*** $\overleftrightarrow{TG} \perp \overline{OT}$

**Secant** – a line that intersects a circle at **two points.**

$\overleftrightarrow{SE}$ ***is a secant line.***




**Inscribed Angle -** an angle made from points sitting on the circle's edge.



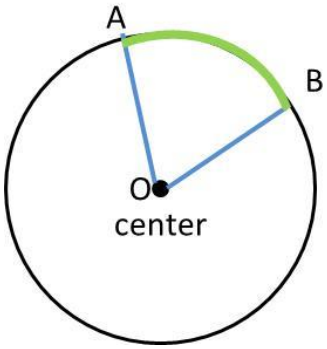

A and C are "end points"
B is the "apex point"

**Central angle-** an angle with vertex at the center of the circle

**Arc** – part of the circumference (edge) of the circle.

The measure of an arc is equal to the measure of

**the central angle formed by its endponts.**




$\overset{\frown}{AB}$ ***is an arc***

$\angle AOB$ ***is a central angle.***

$m\angle AOB = m\overset{\frown}{AB}$

Arcs are named by their endpoints. The dashed arc to the right would be called "arc AB". or "arc BA", the order of the endpoints does not matter. As a shorthand this can be written as the letters AB with a curving line above them

Example: $\overset{\frown}{AB}$ which is read "arc AB".

Notice that this naming can be ambiguous. For example it may mean the major arc AB, where you go the long way around the bottom of the circle. Unless stated otherwise, it always means the minor arc - the shortest of the two.

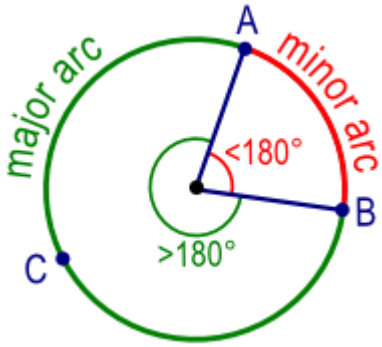
If you want to indicate the major arc, add an extra point and use three letters in the name. For example in the

diagram on the right the major arc is indicated by $\overset{\frown}{ACB}$ which is the long arc from A to B going around the bottom via C.

There are two measures of an arc

1. The length of the arc
2. The angle of the arc

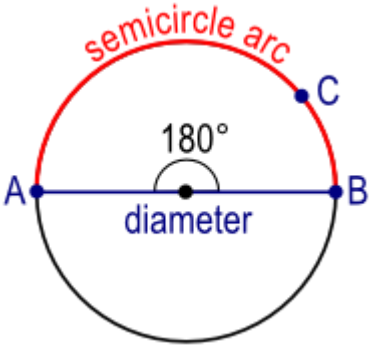
Here is a semicircle arc with a central angle of 180° it covers exactly half of the circumference.The endpoints A and B lie on the diameter of the circle.When naming this arc, we use an extra point C, now we have arc ACB. The third point tells us which half of the circumference the semicircle arc covers. If there are just two points we presume that the named arc is the smallest one on the circumference, the minor arc (as long as the arc is not a semicircle where both arcs are the same size).




**Minor arc – arc whose measure is less that 180 degrees.**

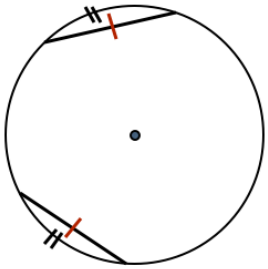
**Major arc – arc whose measure is greater than 180 degrees.**

**Semicircle arc - arc whose measure = 180 degrees.**

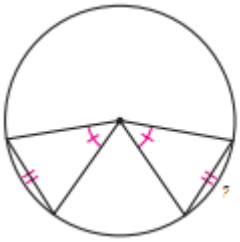


## THEOREMS

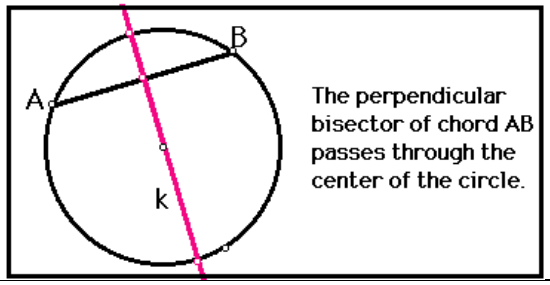
**Chord Central Angles Theorem** If two chords in a circle are congruent, then they determine two central angles that are **congruent.**



**Chord Arcs Theorem** If two chords in a circle are congruent, then their intercepted arcs are **congruent.**

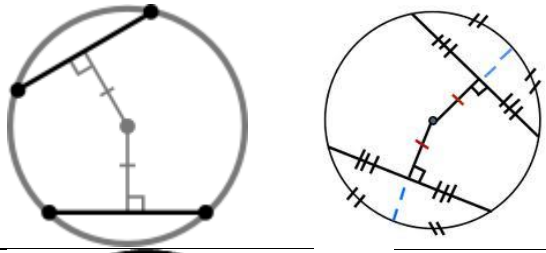


**Perpendicular to a Chord Theorem** The perpendicular from the center of a circle to a chord is the **bisector** of the chord.

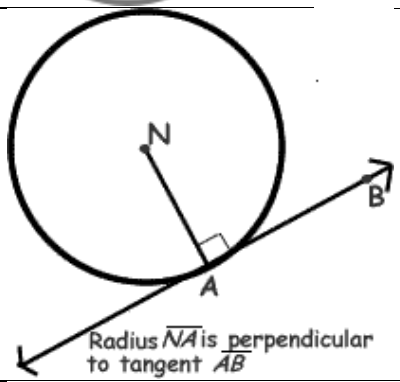



**Chord Distance to Center Theorem** Two congruent chords in a circle are **equidistant** from the center of the circle.

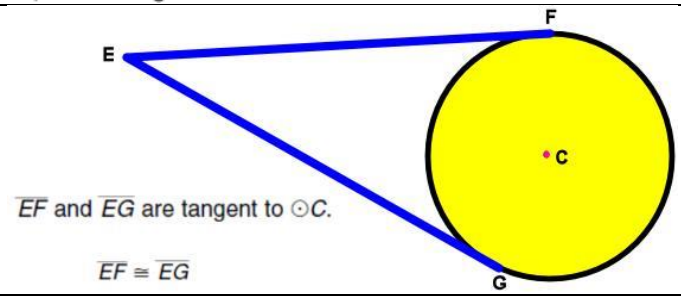
**Perpendicular Bisector of a Chord Theorem** The perpendicular bisector of a chord passes through the **center** of the circle.

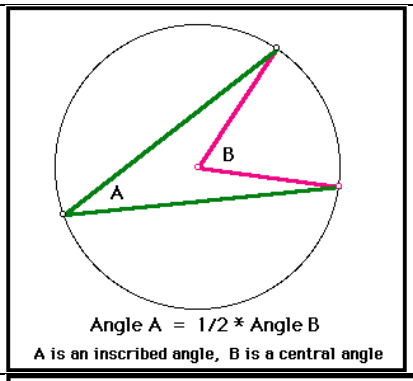
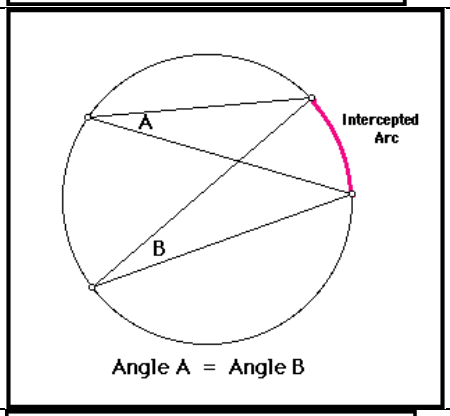
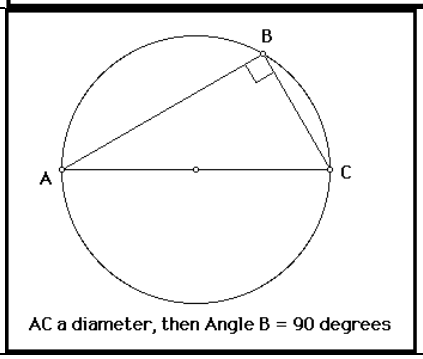
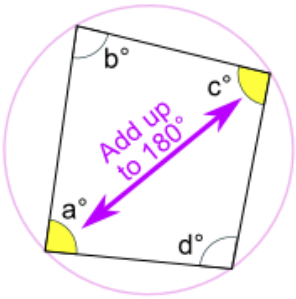
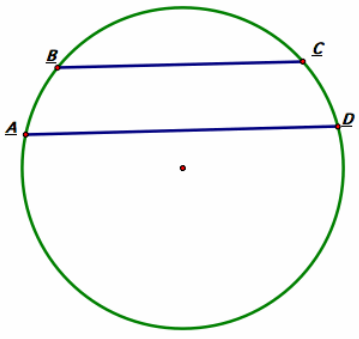


**Tangent Theorem** A tangent to a circle is **perpendicular** to the radius drawn to the point of tangency.




**Tangent Segments Theorem** Tangent segments to a circle from a point outside the circle are **congruent.**



| | |
|---|---|
| **Inscribed Angle Theorem** The measure of an angle inscribed in a circle is **one-half** the measure of the central angle. |  |
| **Inscribed Angles Intercepting Arcs Theorem** Inscribed angles that intercept the same arc are **congruent.** |  |
| **Angles Inscribed in a Semicircle Theorem** Angles inscribed in a semicircle are **right angles**. |  |
| **Cyclic Quadrilateral Theorem** The opposite angles of a cyclic quadrilateral are **supplementary.** |  |
| **Parallel Lines Intercepted Arcs Theorem** Parallel lines intercept **congruent arcs** on a circle. |  If $\overline{BC} \parallel \overline{AD}$, $arcAB \cong arcCD$ |