Example 14

Three points A, B and C are on level ground such that B is due north of A, the bearing of C from A is 046° and the bearing of C from B is 125°. If the distance between A and B is 200 m, calculate the distance of C from A. Give your answer correct to 1 decimal place.
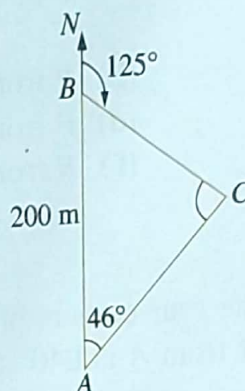
▼ **Solution**

Since the bearing of C from B is 125°,

$$A\hat{B}C = 55°$$
$$\therefore \quad A\hat{C}B = 180° - 46° - 55°$$
$$= 79°$$

Using the Sine Rule, we have

$$\frac{200}{\sin 79°} = \frac{AC}{\sin 55°}$$

$$\therefore \quad AC = \frac{200 \sin 55°}{\sin 79°} = 166.9 \text{ m.}$$

i.e. the distance of C from A is 166.9 m.

Example 15

A boat sailed 20 km from a point P to an island Q, on a bearing of 150°. It then sailed another 30 km on a bearing of 50° to a lighthouse R. Calculate the distance of the lighthouse from P. Give your answer correct to 2 decimal places.
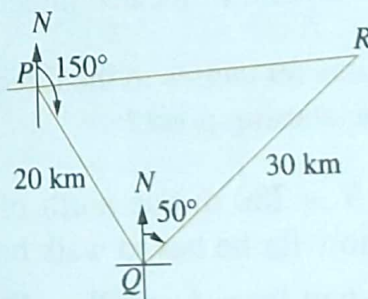
▼ **Solution**

$$P\hat{Q}R = 30° + 50° = 80°$$

Using the Cosine Rule, we have

$$PR^2 = 20^2 + 30^2 - 2 \times 20 \times 30 \times \cos 80°$$
$$= 1\ 091.6$$
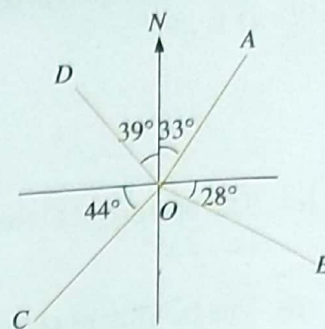$$\therefore \quad PR = 33.04 \text{ km.}$$

i.e. the distance of the lighthouse from P is 33.04 km.

Exercise 10e

1. The figure shows the positions of O, A, B, C and D. State the bearing of
   - (a) A from O;
   - (b) B from O;
   - (c) C from O;
   - (d) D from O.

2. The figure shows the positions of P, A, B and C. State the bearing of
   - (a) A from P;
   - (b) B from P;
   - (c) C from P;
   - (d) P from A;
   - (e) P from B;
   - (f) P from C.
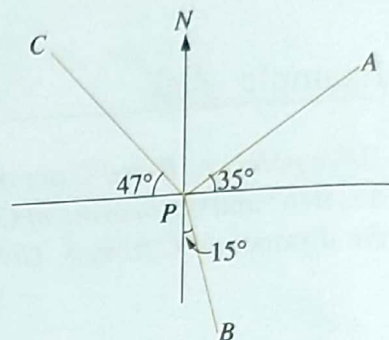
3. The figure shows the positions of P, Q and R. State the bearing of
   - (a) Q from P;
   - (b) P from Q;
   - (c) R from P;
   - (d) P from R;
   - (e) Q from R;
   - (f) R from Q.
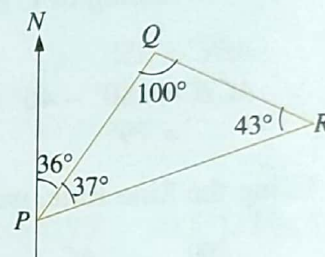
4. A, B, C and D are the four corners of a rectangular plot marked out on level ground. Given that the bearing of B from A is $040°$ and that the bearing of C from A is $090°$, calculate the bearing of
   - (a) B from C;
   - (b) A from C;
   - (c) D from C.

5. A, B and C are three points on level ground. Given that the bearing of B from A is $122°$, $C\hat{A}B = 32°$ and $A\hat{B}C = 86°$, calculate the possible bearing of C from B.

6. P, Q and R are three points on level ground. Given that the bearing of R from P is $135°$, $P\hat{Q}R = 55°$ and $P\hat{R}Q = 48°$, calculate the bearing of
   - (a) P from R;
   - (b) Q from R;
   - (c) P from Q.

7. A point Q is 24 km away and at a bearing of $072°$ from P. From Q, a man walks, at a bearing of $320°$, to a point R, located directly north of P. Calculate the distance of PR and QR.

8. A man swims 50 metres in the direction $045°$, then 60 metres in the direction $145°$. How far is he from the starting point?

9. The point B is 280 m due north of the point A. A man walks from A in the direction $050°$. Calculate how far he has to walk before he is
   - (a) equidistant from A and B;
   - (b) as close as possible to B;
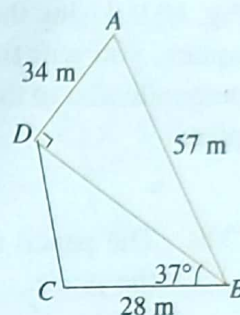   - (c) due east of B.

10. A point P is 12 kilometres due north of another point Q. The bearing of a lighthouse, R, from P is 135° and, from Q, it is 120°. Calculate the distance of PR.

11. Two ships P and Q leave a point at the same time. P sails at 10 km/h on a bearing of 030° and Q sails at 12 km/h on a bearing of 300°. Calculate their distance apart and the bearing of P from Q after 2 hours.

12. A man sails 30 km from a port P to a lighthouse, Q, on a bearing of 128° and then another 25 km to R on a bearing of 295°. Calculate the distance of PR.

13. A, B, C and D are four points on a field. A is due north of D, B is due east of D and $D\hat{B}C = 37°$. Given that $AD = 34$ m, $BA = 57$ m and $BC = 28$ m, calculate

    (a) $B\hat{A}D$;
    (b) the bearing of B from A;
    (c) the area of $\triangle BCD$;
    (d) CD.

14. The diagram represents a map showing a harbour H and three oil rigs, P, Q and R, where R is due east of H. HPQ is a straight line which lies on a bearing of 060° and the angle $HPR = 104°$.
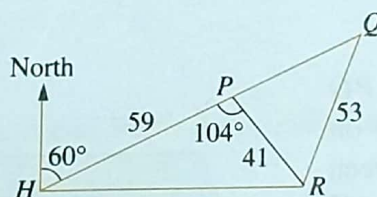
It is given that $HP = 59$ km, $PR = 41$ km and $RQ = 53$ km.

   (a) A supply ship leaves P at 10 45. It sails directly to R, where it stays for 50 minutes, then goes on to Q. When moving, it may be assumed that the ship travels at a constant speed of 12 km/h. At what time does it arrive at Q?
   (b) Calculate the distance HR.
   (c) Calculate the bearing of R from Q.

(C)

14. (a) 1 925   (b) 79.6 km
(c) 191.4°

Exercise 10e *(Pg 226)*

1. (a) 033°   (b) 118°
(c) 226°   (d) 321°
2. (a) 055°   (b) 165°
(c) 317°   (d) 235°
(e) 345°   (f) 137°
3. (a) 036°   (b) 216°
(c) 073°   (d) 253°
(e) 296°   (f) 116°
4. (a) 310°   (b) 270°
(c) 220°
5. 028° or 216°
6. (a) 315°
(b) 003° or 267°
(c) 238° or 032°
7. 34.62 km, 35.51 km
8. 71.12 m
9. (a) 218 m   (b) 180 m
(c) 435.6 m
10. 40.2 km
11. 31.24 km, 080.2°
12. 7.97 km
13. (a) 53.4°   (b) 126.6°
(c) 385.5 m²   (d) 28.8 m

350

(a)

(6)

bearing of D from

(a) bearing of B from C $= 360° - 45° = 315°$

$= 360° - 050° = 310°$

(b) bearing of Q from

(b) bearing of A from C $R = 003°$

$= 270°$

(c) bearing of P from Q

(c) bearing of D from C

$= 180 + 090 = 220°$

$= 180 + 090 = 220°$

(5)

(a)
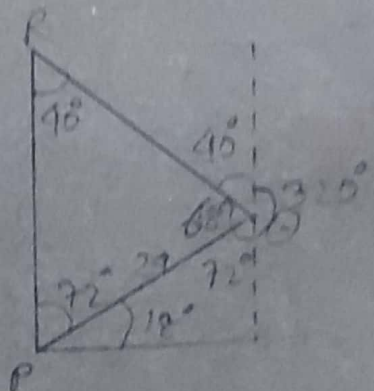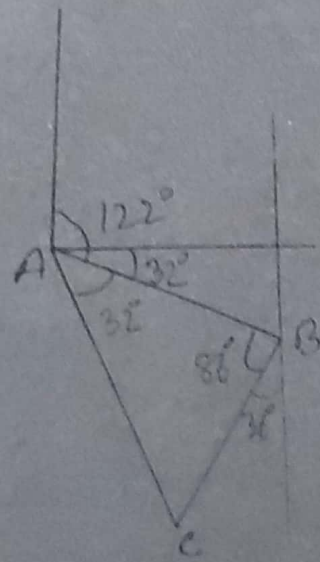
$$\frac{PQ}{\sin 40°} = \frac{PR}{\sin 68°} = \frac{QR}{\sin 72°}$$

bearing of C from B

$$\frac{24}{\sin 40°} = \frac{PR}{\sin 68°} = \frac{QR}{\sin 72°}$$

$180 + 036 = 216°$

$PR = \frac{24 \times \sin 68°}{\sin 40°} = 34.6$

$QR = \frac{24 \times \sin 72°}{\sin 40°} = 35.5$

⑧

He in $= \sqrt{50^2 + 60^2 - 2 \times 50 \times 60 \times \cos 9^\circ}$

$= \sqrt{5058.11}$

$= 71.12$ m from

The starting point.

$\dfrac{PR}{\sin 60^\circ} = \dfrac{12}{\sin 15^\circ}$

$\Rightarrow PR = \dfrac{12 \times \sin 60^\circ}{\sin 15^\circ}$

$\Rightarrow PR = 40.2$ m

⑪

Their distance PQ

$= \sqrt{20^2 + 24^2 - 2 \times 20 \times 24 \times \cos 90^\circ}$

$= \sqrt{400 + 576 - 0}$

$= \sqrt{976}$

$= 31.24$ km

$\dfrac{PQ}{\sin OQP} = \dfrac{PQ}{\sin 90^\circ}$

$\Rightarrow \dfrac{20}{\sin OQP} = \dfrac{31.24}{\sin 90^\circ}$

$\Rightarrow \sin OQP = \dfrac{20}{31.24}$

$\Rightarrow O\hat{Q}P = \sin^{-1}(0.64)$

$\Rightarrow O\hat{Q}P = 39.8^\circ$

bearing of P from Q

$= 180^\circ - 039.8 - 060^\circ$

$= 080.2^\circ$. A

(12)

Their distance PS
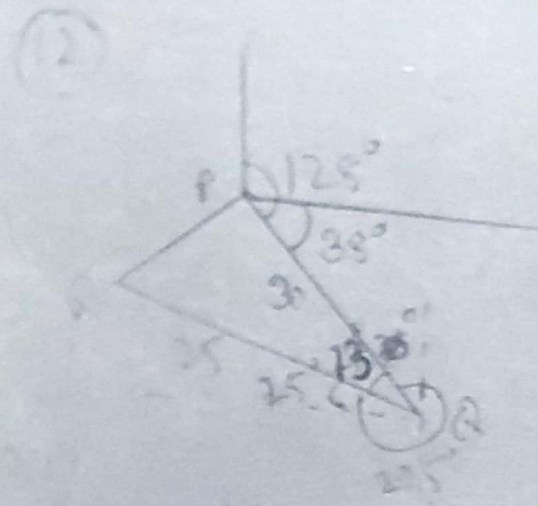
$$= \sqrt{25^2 + 30^2 - 2 \times 25 \times 30 \times \cos 13^\circ}$$

$$= 7.97 \text{ km}.$$

:D

(13)

@ $BD^2 = AB^2 - AD^2$

$\Rightarrow BD = \sqrt{57^2 - 34^2}$

$\Rightarrow BD = 45.7$

$\cos B\hat{A}D = \dfrac{34^2 + 57^2 - 45.7^2}{2 \times 34 \times 57}$

$\Rightarrow B\hat{A}D = \cos^{-1}(0.60)$

$\Rightarrow B\hat{A}D = 53.3^\circ$

ⓑ area of $\triangle BCD$

$= \frac{1}{2} \times 28 \times 45.7 \times \sin 37^\circ$

$= 385.09$